GCE Examinations

Further Pure Mathematics Module FP1

Advanced Subsidiary / Advanced Level

Paper H

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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1. *(a)* Given that

$$\mathrm{f}(r) = r!\,,$$

show that

$$\mathrm{f}(r+1) - \mathrm{f}(r) = r \times r! \qquad \textbf{(2 marks)}$$

(b) Hence find

$$\sum_{r=1}^{n} (r \times r!) \qquad \textbf{(4 marks)}$$

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2. *(a)* Given that

$$y = \frac{2x}{x^2 + 9},$$

express x in terms of y. **(5 marks)**

(b) Hence prove that for all real values of x

$$-\frac{1}{a} \leq \frac{2x}{x^2 + 9} \leq \frac{1}{a},$$

where a is a positive integer which you should find. **(3 marks)**

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3. Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + xy = 1 - y,$$

giving your answer in the form $y = \mathrm{f}(x)$. **(9 marks)**

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4.

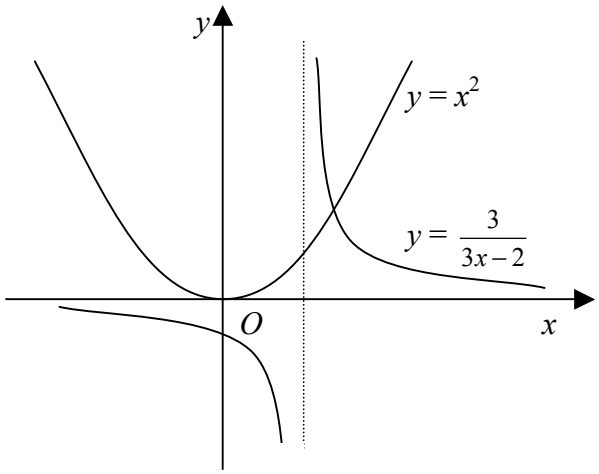


Fig. 1

Figure 1 shows part of the curves $y = x^2$ and $y = \dfrac{3}{3x-2}$.

The curves meet at the point with x-coordinate α.

(a) Find the integer N such that $\dfrac{N}{10} < \alpha < \dfrac{N+1}{10}$. **(4 marks)**

(b) Use interval bisection on the interval found in part *(a)* to find the value of α correct to 2 decimal places. **(5 marks)**

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5. Given that

$$\mathrm{f}(z) \equiv z^4 - 4z^3 + kz^2 - 4z + 13,$$

where k is a real constant, and that $z = \mathrm{i}$ is a solution of the equation $\mathrm{f}(z) = 0$,

(a) show that $k = 14$, **(3 marks)**

(b) find all solutions of the equation $\mathrm{f}(z) = 0$. **(7 marks)**

Turn over

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6. The shape of a company logo is to be the region enclosed by the curve with polar equation

$$r^2 = a^2 \sin 2\theta, \quad 0 \leq \theta \leq \tfrac{\pi}{2}.$$

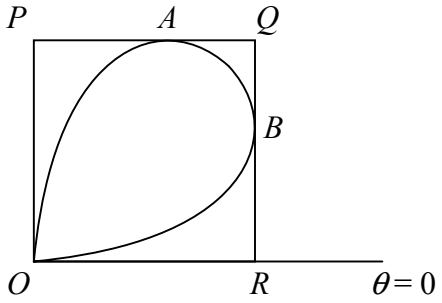


Fig. 2

A sign in the shape of the logo is to be made by cutting the area enclosed by the curve from a square sheet of metal $OPQR$ where O is the pole and R lies on the initial line, $\theta = 0$, as shown in Figure 2. PQ and QR are tangents to the curve, parallel and perpendicular to the initial line respectively, at the points A and B on the curve.

(a) Find the value of θ at the point A. **(7 marks)**

(b) Show that the area of $OPQR$ is $\dfrac{3\sqrt{3}}{8}a^2$. **(3 marks)**

(c) Find the area of the metal sheet which is not used. **(5 marks)**

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7. Given that $x = ke^{-t}$ satisfies the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 8e^{-t},$$

(a) find the value of k. **(3 marks)**

(b) Hence find the solution of the differential equation for which $x = 1$ and $\dfrac{dx}{dt} = 3$ at $t = 0$.

(8 marks)

The maximum value of x occurs when $t = T$.

(c) Show that the maximum value of x is $\frac{40}{27}$ and find the value of T. **(7 marks)**

END

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